Radial flow towards well in leaky unconfined aquifer

Phoolendra K. Mishra[1] (pkmishra@fullerton.edu),Velimir. V. Vesselinov[2], Kristopher. L. Kuhlman[3]
[1]Department of Civil & Environmental Engineering, California State University, Fullerton, CA
[2]EES-16, Los Alamos National Laboratory, Los Alamos NM,
[3]Repository Performance Department, Sandia National Laboratories, Carlsbad, NM

Summary

- Drawdown due to pumping a large-diameter (e.g., water supply) well in an unconfined aquifer is affected by wellbore storage [Papadopulos and Cooper Jr, 1967].
- Narasimhan and Zhu [1993] used a numerical model to demonstrate that early time drawdown in an unconfined aquifer tends to be dominated by wellbore storage effects.
- Mishra and Neuman [2011] developed an analytical unconfined solution, which considers both pumping-well wellbore storage capacity, and three-dimensional axi-symmetrical unsaturated zone flow.
- Malama et al. [2007] developed a solution for three-dimensional aquitard flow in a finite thickness aquitard, but considered the zero-radius pumping well to be fully penetrating and ignored the flow in unsaturated zone.
- Here, we develop a more general leaky-unconfined aquifer solution by considering a partially penetrating large-diameter well and including the effects of unsaturated zone flow following Mishra and Neuman [2011].
- The solution is used to investigate the effect of an aquitard on drawdown in overlying unconfined aquifer.
- We conclude by investigating the effects of wellbore storage capacity and the unsaturated zone on drawdown observed in the aquitard.

Statement of Problem

- We consider an infinite radial compressible unconfined aquifer above a finitely thick aquitard (Figure 1).
- The aquifer and aquitard are each spatially uniform, homogeneous and anisotropic, with constant specific storage S_s and S_{s1}, respectively.
- Following Mishra and Neuman [2011], drawdown in the saturated zone can be expressed as

$$s = s_C + s_U \quad (1)$$

where s_C is solution for flow to a partially penetrating well of finite radius in a confined aquifer and s_U is a correction accounting for the underlying aquitard, water table and unsaturated zone effects.

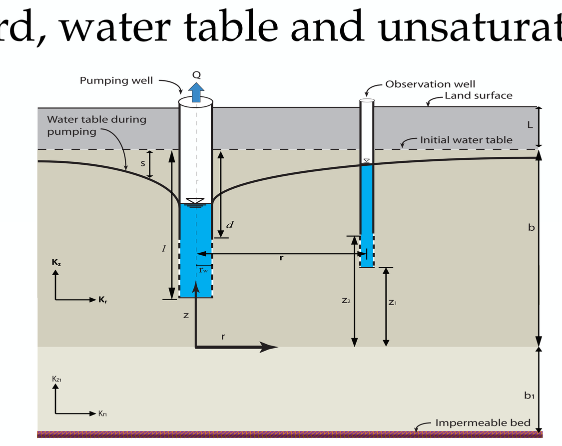

FIG. 1: Schematic representation of finite radius pumping well in a leaky unconfined aquifer- aquitard system

Results and Discussion

- We extend the theory of leakage in unconfined aquifers by (1) including water flow and storage in the unsaturated zone above the water table, and (2) allowing the finite-diameter pumping well to partially penetrate the aquifer.
- The Laplace transformed $\bar{s}_U$ derived as

$$\bar{s}_U(r_D, z_D, p_D) = \int_0^\infty \left(\rho_1 e^{\mu z_D} + \rho_2 e^{-\mu z_D}\right) \frac{r_D^2 K_D}{r^2} y J_0\left[y K_D^{1/2} r_D\right] \mathrm{d}y \quad (2)$$

where $\rho_1 = \frac{\left(\frac{\mu}{qb}+1\right)e^{-\mu}(\bar{s}_c)_{z_D=0} - \left(\frac{\mu}{q_1 b}+1\right)e^{-\mu}(\bar{s}_c)_{z_D=1}}{\Delta}$, $\rho_2 = \frac{\left(\frac{\mu}{qb}-1\right)e^{-\mu}(\bar{s}_c)_{z_D=0} - \left(\frac{\mu}{q_1 b}-1\right)e^{-\mu}(\bar{s}_c)_{z_D=1}}{\Delta}$, $q_1 b = R_{K_z}\mu_1 \tanh(\mu_1 R_b)$, $\mu_1^2 = \frac{y^2}{R_{K_D}} + \frac{p_D}{t_s K_D r_D^2 R_{K_D} R_{\alpha_S}}$, $R_{K_D} = K_{D1}/K_D$, $R_{K_z} = K_{z1}/K_z$, $R_{\alpha_s} = \alpha_{S1}/\alpha_s$, $R_b = b_1/b$, $\alpha_{s1} = K_{r1}/S_{s1}$, and $\Delta = \left(\frac{\mu}{qb}+1\right)\left(\frac{\mu}{q_1 b}-1\right)e^{-\mu} - \left(\frac{\mu}{qb}-1\right)\left(\frac{\mu}{q_1 b}+1\right)e^{\mu}$.

- The Laplace transformed aquitard drawdown derived as

$$\bar{s}_1(r_D, z_D, p_D) = \int_0^\infty \frac{(\bar{s}_c)_{z_D=0} + \rho_1 + \rho_2}{\cosh(\mu_1 b_1/b)} \cosh\left[\mu_1(z_D + R_b)\right] \times \frac{r_D^2 K_D}{r^2} y \mathrm{J}_0\left[y K_D^{1/2} r_D\right] \mathrm{d}y \quad (3)$$

- The time domain equivalents s_C, s_U, s_1 and σ of $\bar{s}_C$, $\bar{s}_U$, $\bar{s}_1$ and $\bar{\sigma}$ are obtained through numerical Laplace transform inversion.

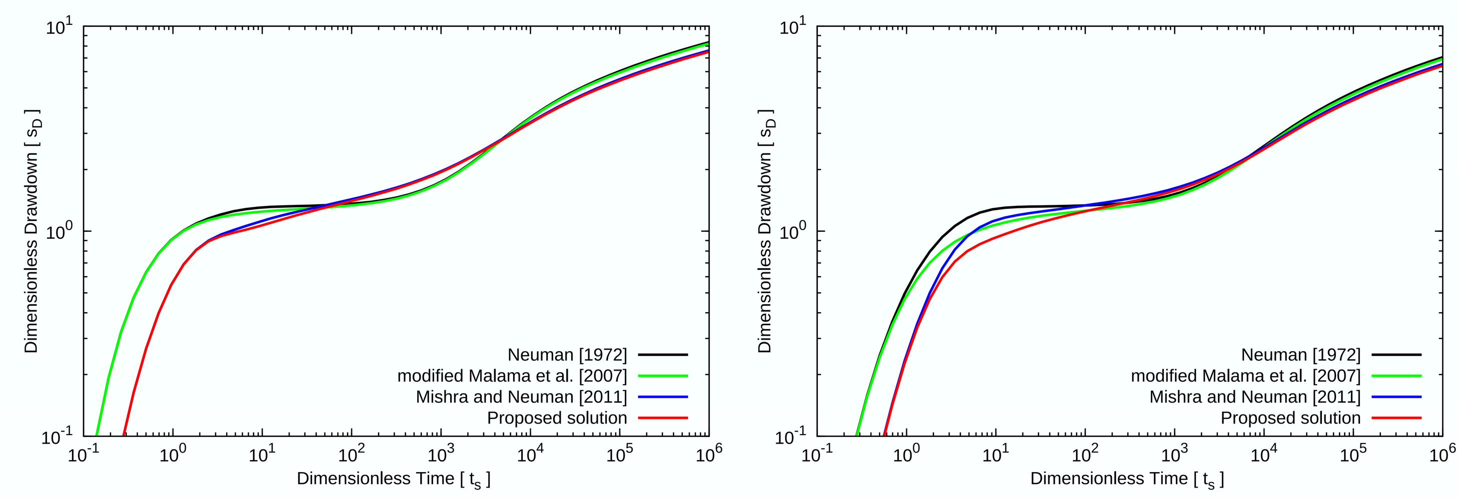

FIG. 2: 2a (left) near water table , 2(b) far from water table

- Figures 2a and 2b compare variations in dimensionless drawdown with dimensionless time at $z_D = 0.75$ and $z_D = 0.25$ predicted by our proposed solution and the solutions of Mishra and Neuman [2011], Neuman [1972], and the modified solution of Malama et al. [2007].

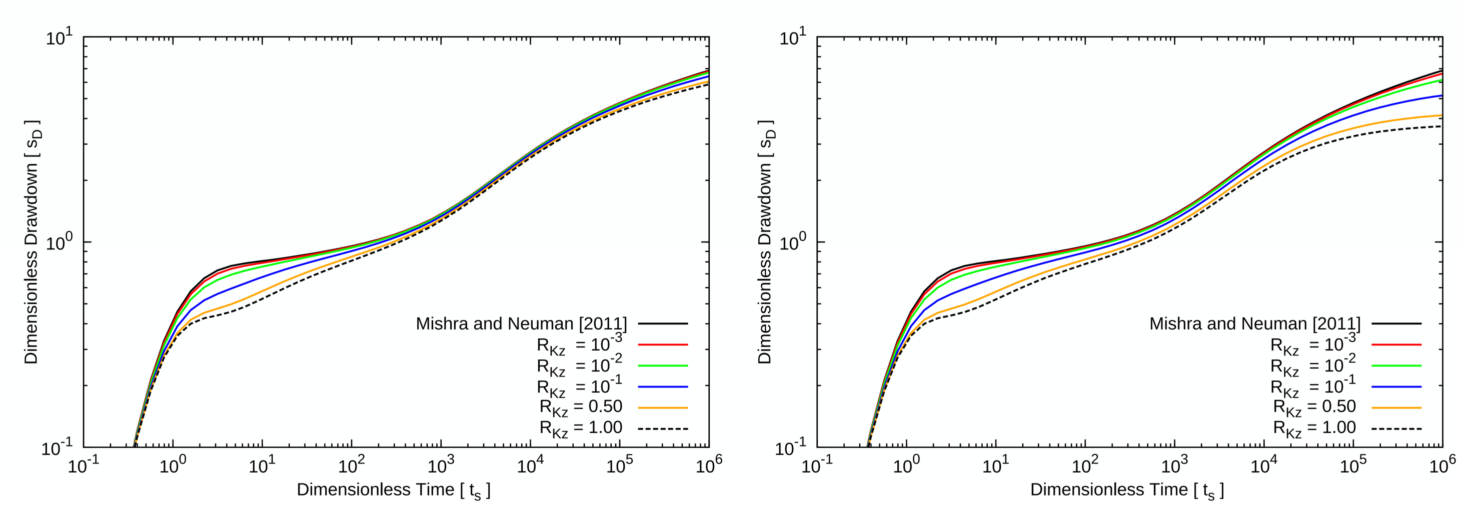

FIG. 3: 3a (left) small R_{k_r}, 3(b) right large R_{k_r}

- Unsaturated zone effects are often more important than leakage effects when the observation location is close to the water table.
- Figures 3a and 3b show dimensionless time-drawdown variations at dimensionless radial distance $r_D = 0.5$ and dimensionless unconfined aquifer saturated zone elevation $z_D = 0.25$ with different values of $R_{K_z} = K_{z1}/K_z$ when the radial aquitard hydraulic conductivity is small ($R_{K_r} = K_{r1}/K_r = 10^{-6}$) and large ($R_{k_r} = 1.0$)
- Figure 4 depicts the effect that changes in a_{kD}, the dimensionless relative hydraulic conductivity exponent, have on dimensionless time-drawdown at dimensionless radial distance $r_D = 0.2$ and dimensionless aquitard elevation $z_D = -0.25$.

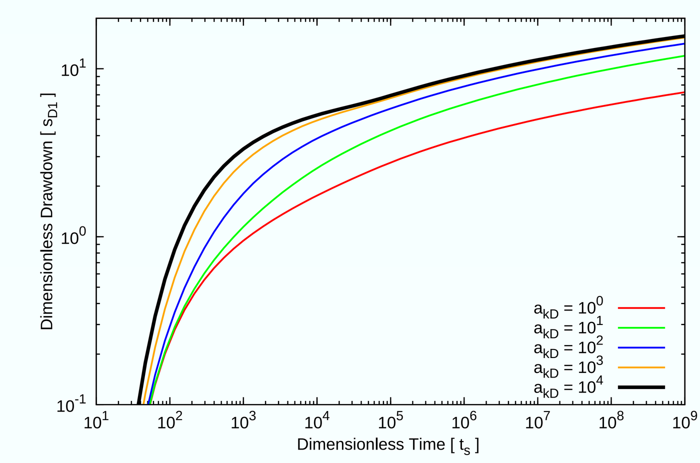

FIG. 4: Effects of hydraulic conductivity exponent

- Figure 5 shows the effects that changes in a_{cD}, the dimensionless effective saturation exponent, have on dimensionless time-drawdown at dimensionless radial distance $r_D = 0.2$ and dimensionless aquitard elevation $z_D = -0.25$.

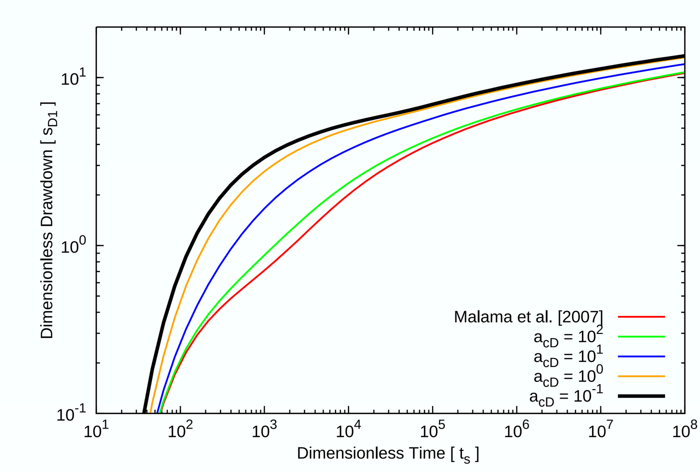

FIG. 5: Effects of effective saturation exponent

- The unsaturated zone properties not only affect the unconfined aquifer time-drawdown behavior but they also impact the observed aquitard response.

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References

B. Malama, K. Kuhlman, and W. Barrash. Semi-analytical solution for flow in leaky unconfined aquifer-aquitard systems. *Journal of Hydrology*, 346(1-2):59–68, 2007. ISSN 0022-1694.

P. Mishra and S. Neuman. Saturated-unsaturated flow toward a well with storage in a compressible unconfined aquifer. *Water Resources Research*, 86(7):W12508, 2011. ISSN 0043-1397.

T. Narasimhan and M. Zhu. Transient flow of water to a well in an unconfined aquifer: applicability of some conceptual models. *Water Resources Research*, 29(1):179–191, 1993.

S. Neuman. Theory of flow in unconfined aquifers considering delayed response of the water table. *Water Resources Research*, 8(4):1031–1045, 1972. ISSN 0043-1397.

I. Papadopulos and H. Cooper Jr. Drawdown in a well of large diameter. *Water Resources Research*, 3 (1):241–244, 1967.