Transient AEM for Aquifer Test Analysis

using the Laplace transform

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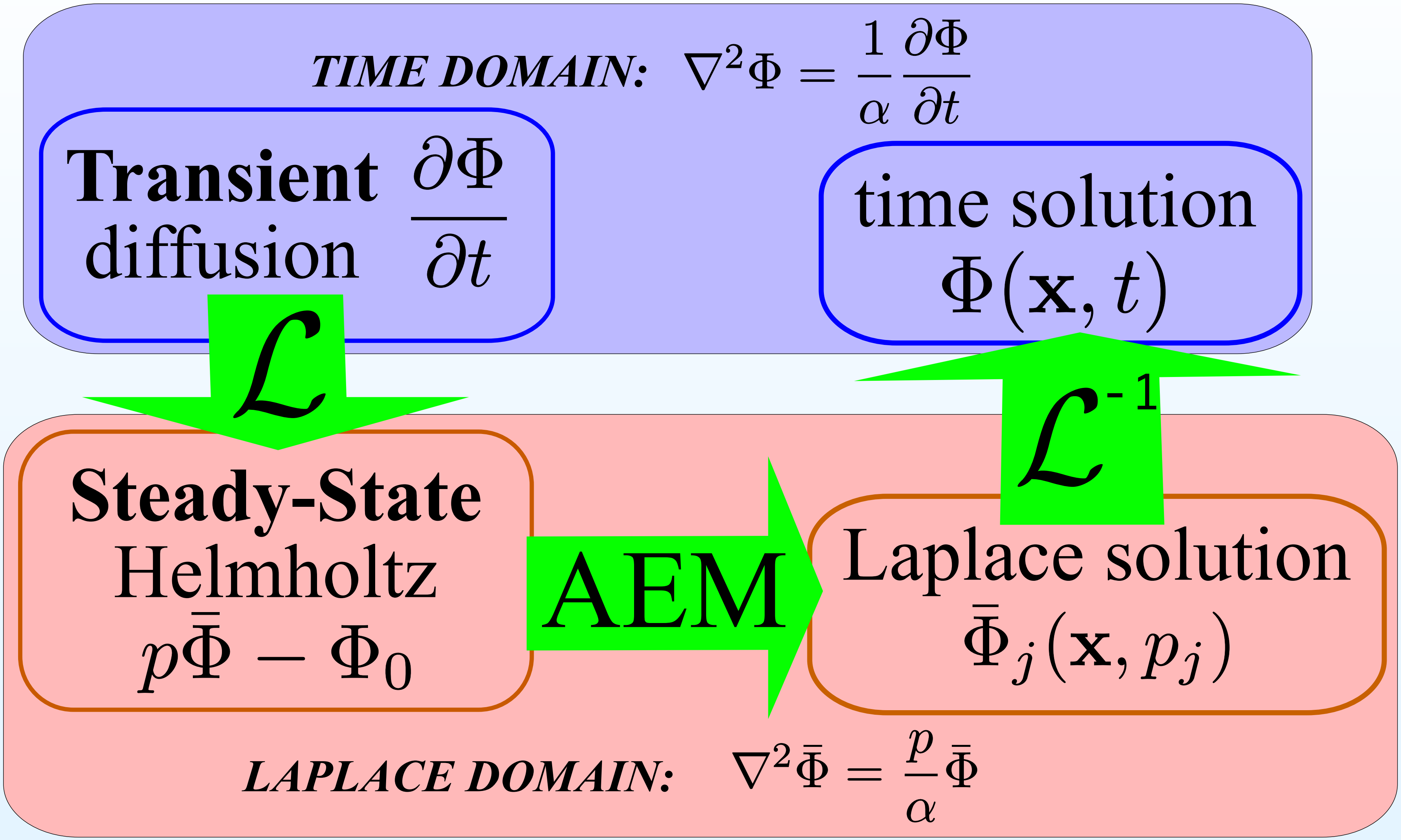
Laplace transform AEM



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general source term

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- use $\mathcal{L}/\mathcal{L}^{-1}$ methods & tools for sources

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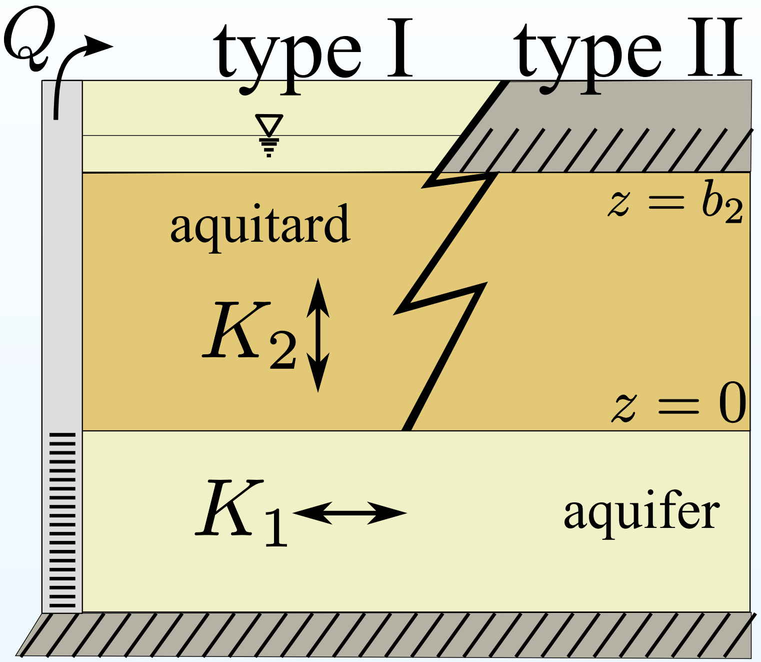

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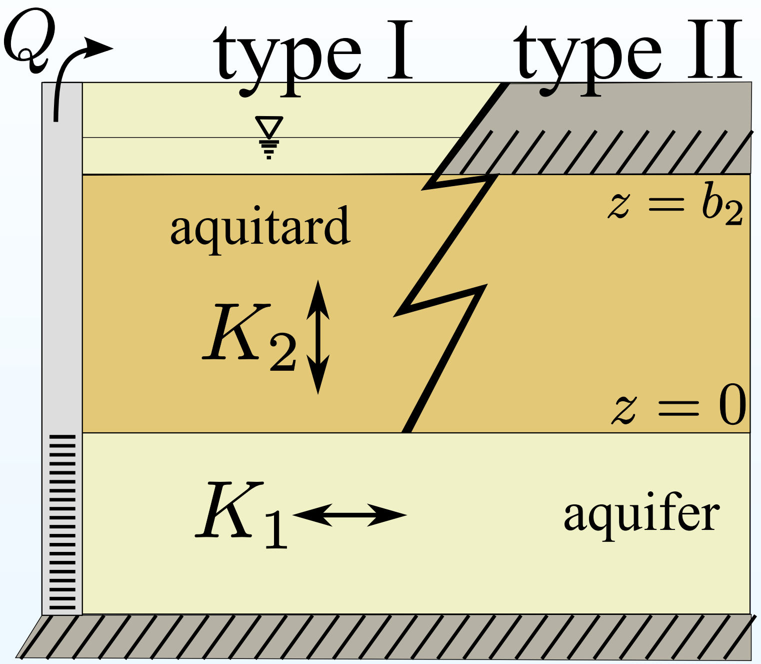

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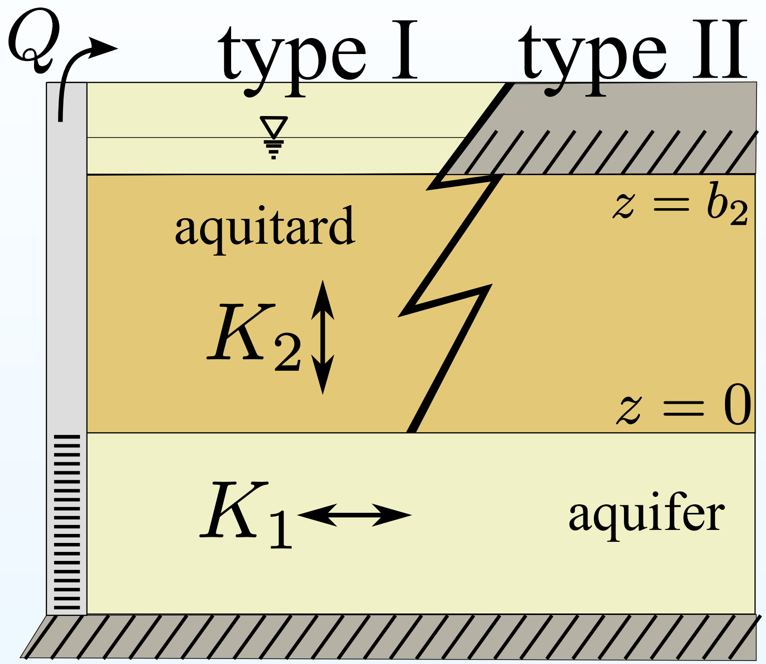

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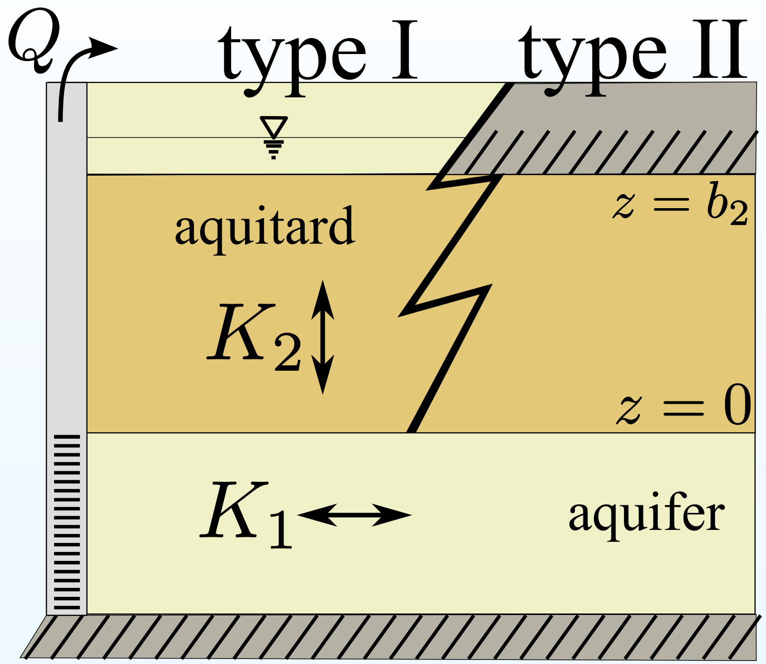

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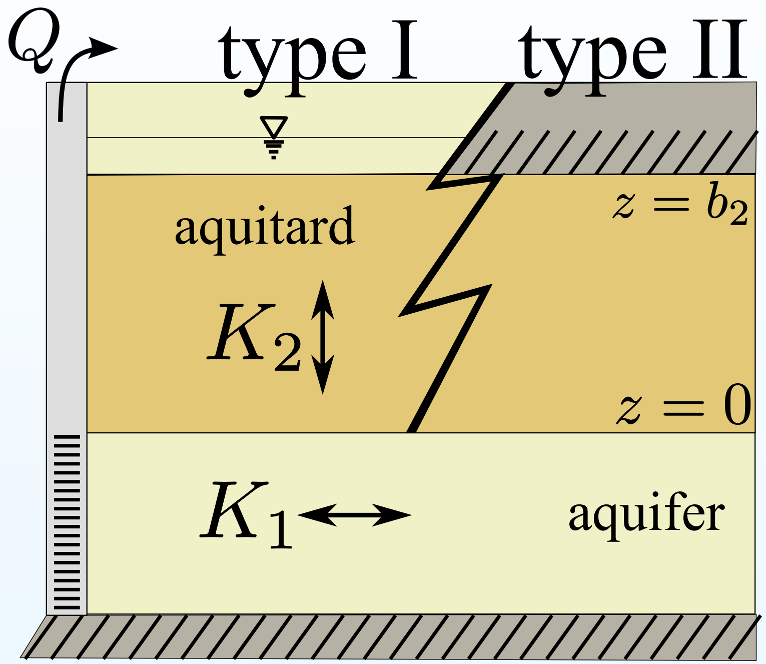

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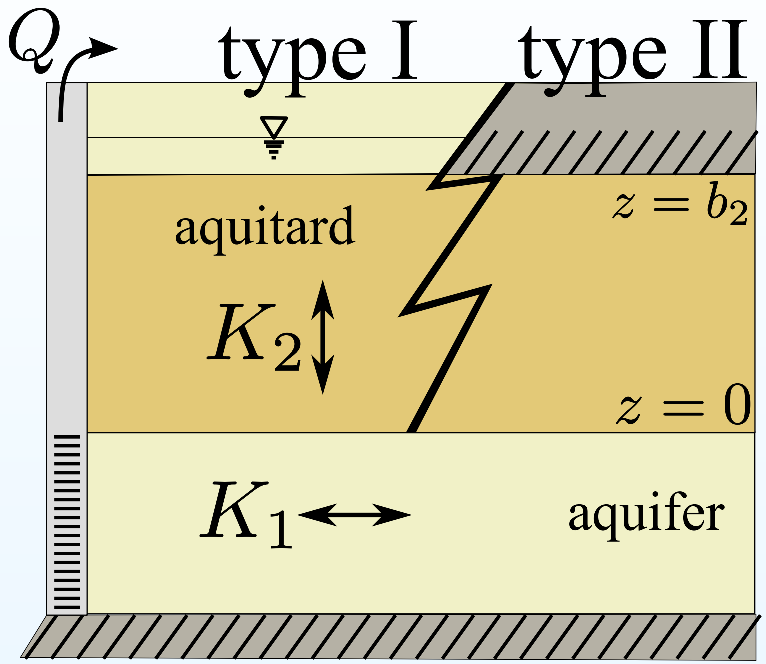

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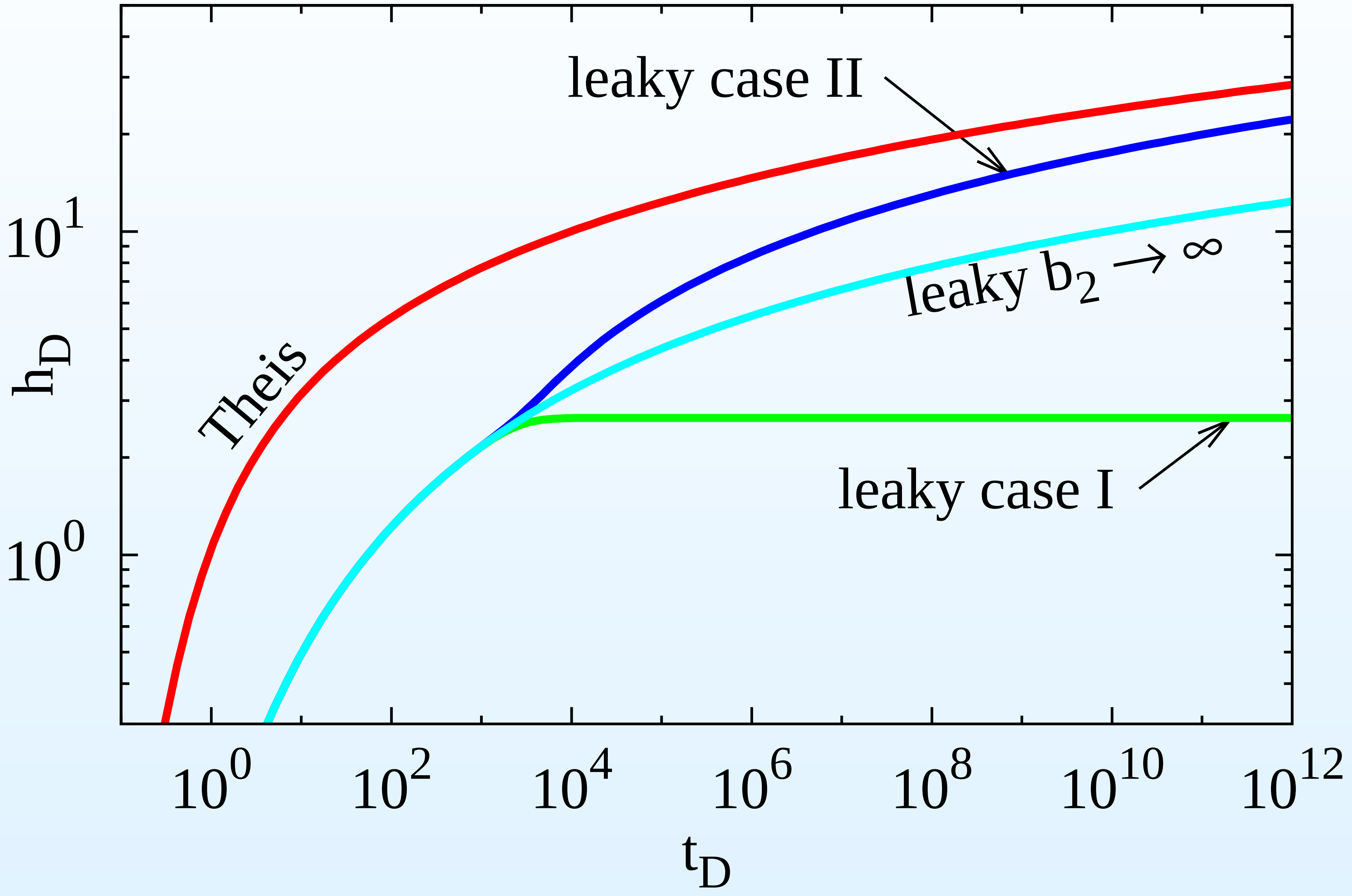
effects of aquifer storage and aquitard leakage

Ex 1: Leaky point source example



Ex 2: Boulton's Delayed Yield (1954)

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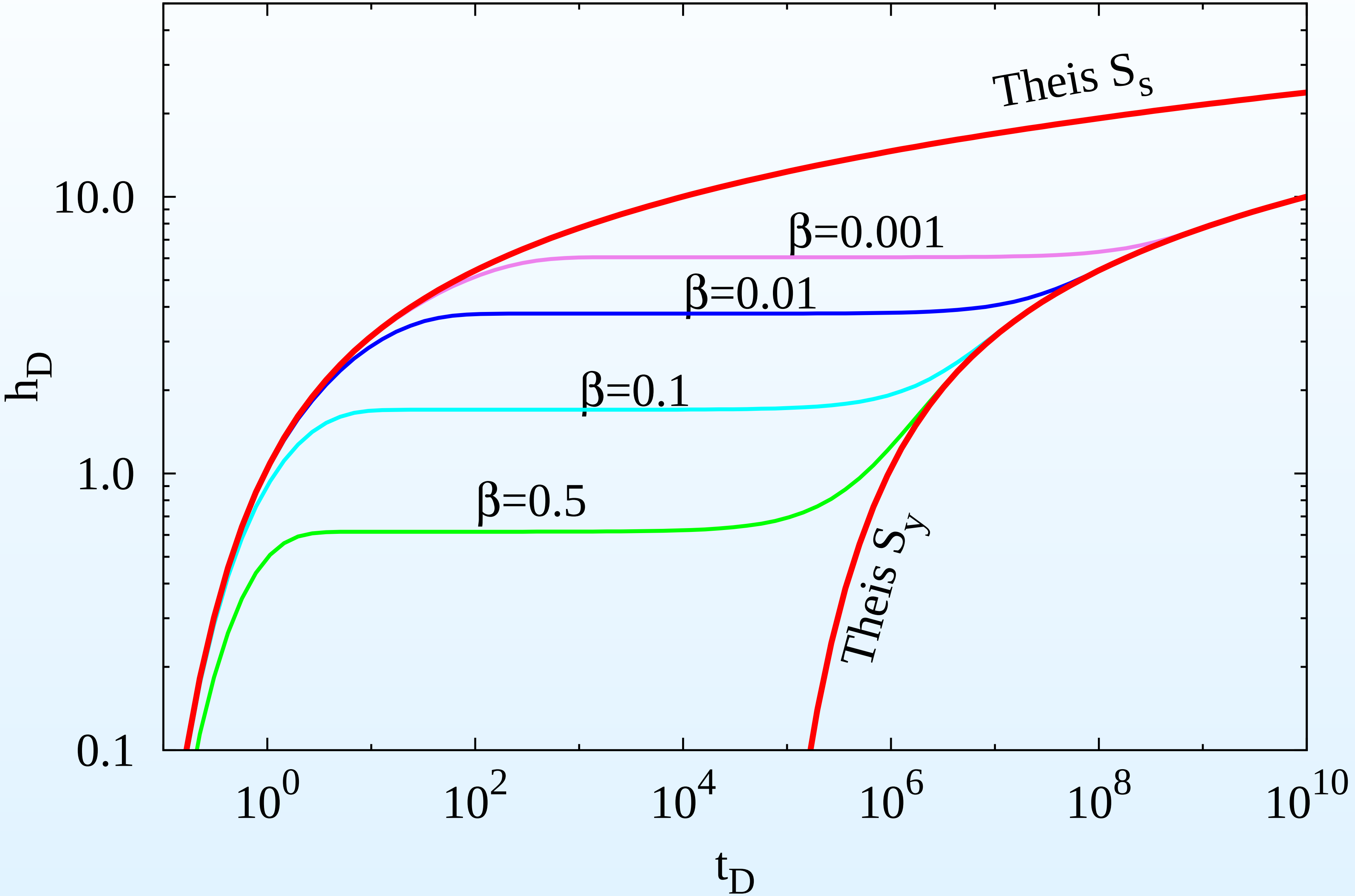

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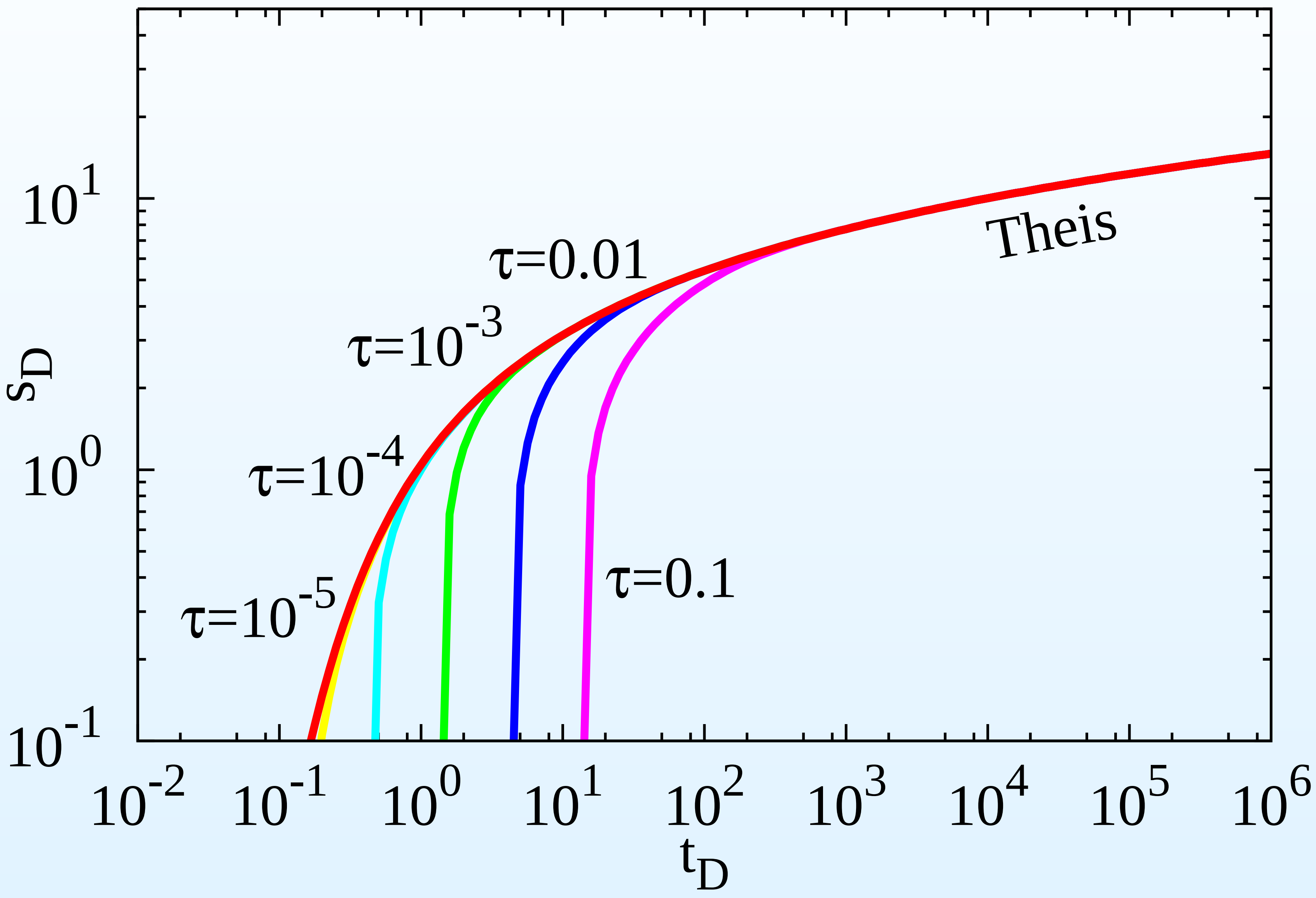
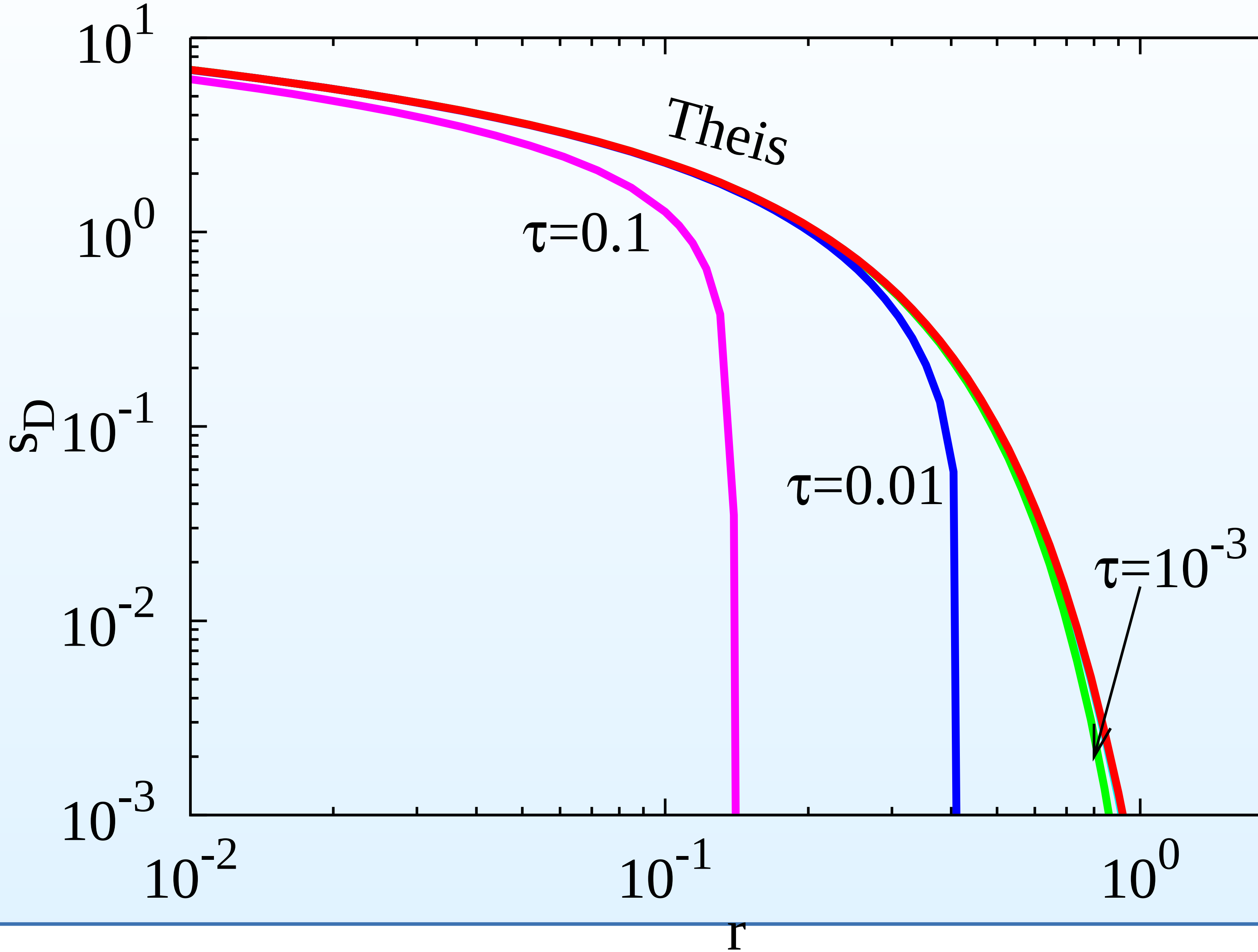
$$\nabla^2\Phi = \frac{1}{\alpha}\left[\frac{\partial\Phi}{\partial t} + \tau\frac{\partial^2\Phi}{\partial t^2}\right]$$

Ex 3: Time drawdown (inertia)



Ex 3: Distance drawdown (inertia)



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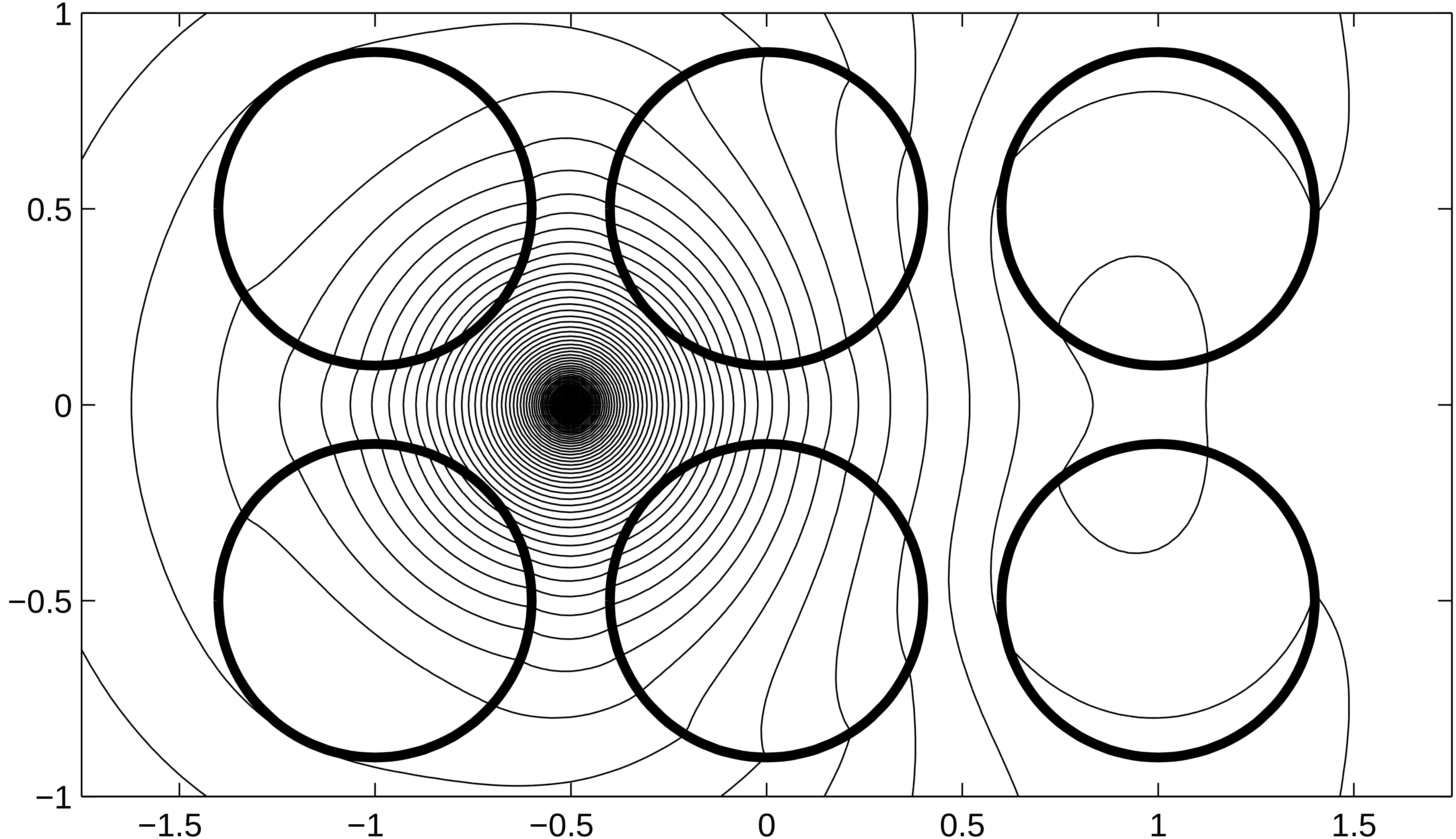
- **LT-AEM extends AEM to aquifer test analysis**


1
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0
−0.5
−1
−1.5
−1
−0.5
0
0.5
1
1.5

Thank You