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Multi-Porosity Transport of Natural Tracers in a Fractured System

Kristopher L. Kuhlman[1], Bwalya Malama, Jason E. Heath, W. Payton Gardner, and David G. Robinson

[1]klkuhlm@sandia.gov; AGU Fall Meeting 2013 H53A-1405 Sandia National Laboratories†, Albuquerque and Carlsbad, NM; SAND2013-10429P

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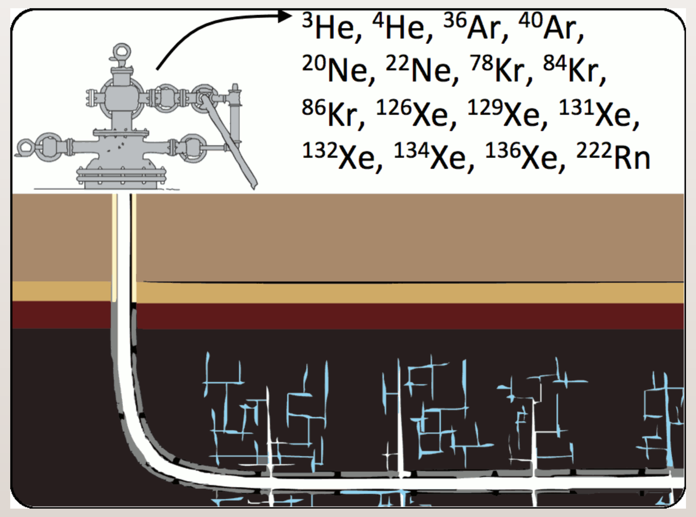
Predicting meaningful production decline in non-traditional shale oil and gas wells is a complex problem. We are developing a Bayesian framework to evaluate the use of naturally occurring noble gas tracers in fractured shale hydrocarbon wells. Using noble gas tracers along with traditional production analysis methods will increase the information content produced from oil and gas wells in low-permeability shale formations. The Bayesian framework will allow the use of multiple predictive models to interpret observed production declines in producing oil and gas wells, including engineering approaches, semi-analytic diffusion models, and non-linear multiphase numerical flow models.



Helium and other noble gases exist in high concentrations in hydrocarbon reservoirs, as by-products of radioactive decay of naturally occurring uranium and thorium. Helium is a very small molecule, and all noble gases are chemically inert (they do not sorb to the formation like methane does). These properties make He an ideal gas to sample during hydrocarbon production to obtain more information regarding:

- early detection of reservoir and well/completion flow regimes; and
- understanding of transport and delivery mechanisms inside the reservoir.

We are combining approximate solutions, semi-analytical solutions, and numerical models to predict flow of noble gases from the reservoirs to horizontal wells completed using hydraulic fracturing.

Naturally occurring tracers may only be brought to the borehole through advection (since they have a uniform initial concentration), driven by pressure gradients. Different species may have their own effective permeabilities, porosities, and compressibilities – helium being the most favorable, because it is a small inert molecule.

Multiporosity Flow Model

We take the classical *double porosity* flow model of Warren and Root [1963] (W-R), and extend it using the *multirate transport* approach of Haggerty and Gorelick [1995]. The dimensionless governing equation for fracture flow (subscript f) can be written as

$$\omega \frac{\partial \psi_f}{\partial t_D} + \sum_{j=1}^{N} \left[\omega_j \rho_j \frac{\psi_j}{\partial t_D} \right] = \nabla^2 \psi_f, \qquad (1)$$

where ψ_f is the dimensionless change in pressure, $\omega_j = \frac{c_j \phi_j}{c_f \phi_f + \sum c_j \phi_j}$ is a ratio of fracture or matrix to total storage capacity (c is compressibility and ϕ is porosity), $t_D = t/(r_w^2 \mu \phi c / k_f)$ is dimensionless time, μ is viscosity, r_w is wellbore radius, and ρ is a probability distribution function. The dimensionless matrix flow equation (subscript j) for pseudo-steady interporosity flow is

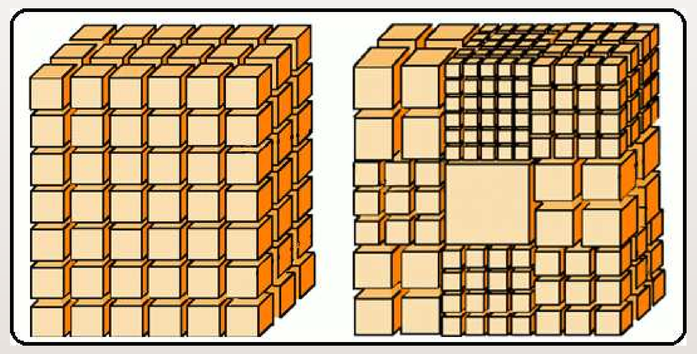
$$\frac{\partial \psi_j}{\partial t_D} = \lambda_j \left[\psi_f - \psi_j \right] \qquad j = 1, \dots, N, \qquad (2)$$

where $\lambda_j = k_j/(k_f \omega_j)$ is a interporosity flow coefficient. Using the Laplace transform to solve the matrix and fracture governing equations, we get:

$$\omega s \bar{\psi}_f + s \sum_{j=1}^{N} \left[\omega_j \rho_j \frac{\lambda_j}{s + \lambda_j} \right] \bar{\psi}_f = \nabla_D^2 \bar{\psi}_f, \qquad (3)$$

where s is the Laplace parameter and an overbar indicates a Laplace-transformed variable. The solution is computed using a numerical inverse Laplace transform. The ρ_j and λ_j coefficients can be chosen in different manners to obtain different types of solutions.

The multiporosity model is sufficiently flexible to represent flow in several different classically considered cases as degenerate or special cases. Using a flow model with a great deal of flexibility allows the modeler more options during the parameter estimation portion of the analysis. Pseudo-steady-state double porosity W-R flow can be simulated using a matrix governing equation with just a single term in (2) ($N = 1$). Triple porosity or further extensions are handled in an analogous manner ($N = 2$, etc.).



double porosity (L) and multi-porosity (R)

The transient double interporosity flow model of Kazemi [1969] can be approximated using an infinite sum of pseudo-steady matrix porosities, using an approach similar to Haggerty and Gorelick [1995]. The coefficients in the solution can be chosen as

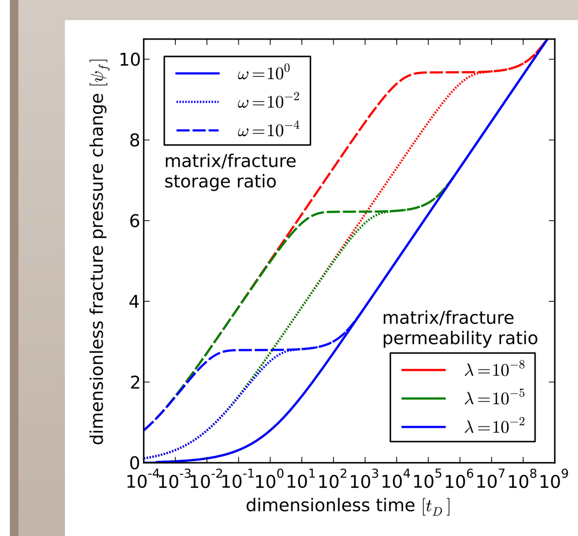
$$\lambda_i = \frac{(2i-1)^2 \pi^2 \gamma_i}{4} \qquad (4)$$

$$\rho_i = \frac{8}{(2i-1)^2 \pi^2} \qquad (5)$$

This distribution of pseudo-steady interporosity flow of the simple W-R dual-porosity case when $N = 1$, to the transient interporosity flow of the Kazemi model for $N \to \infty$ (see well test solution below for examples).

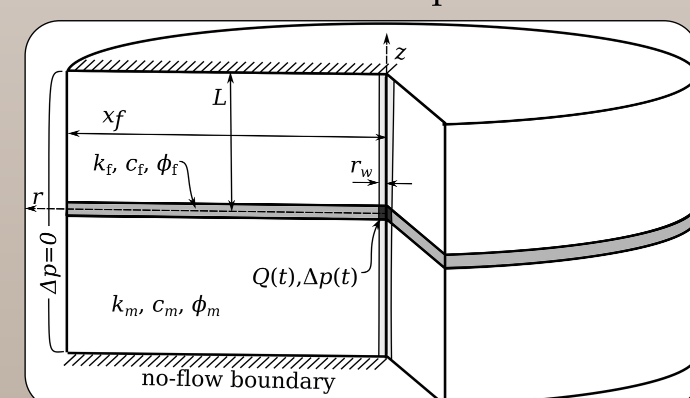
Radial Well Test Solutions

The W-R pressure solutions for pseudo-steady double-porosity interporosity flow are shown for a range of permeability and storage coefficient ratios. Solutions which assume radial symmetry are associated with the fracture/matrix geometry shown in the diagram below.



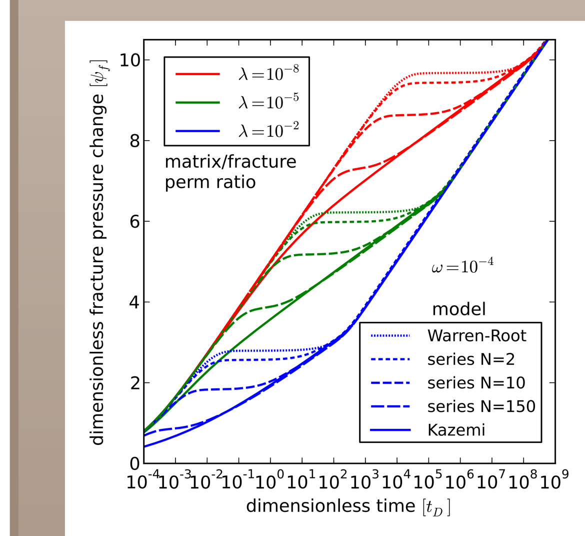
Warren & Root (W-R) (above)

The well is open to a man-made fracture, which is planar and centered on the production well. A multi-stage fracked well consists of several parallel fractures.



Radially symmetric well test solution geometry

The upper figure on the left shows the variation in the traditional W-R solution over ranges of λ and ω, representing permeability and storage ratios between the matrix and fracture systems.



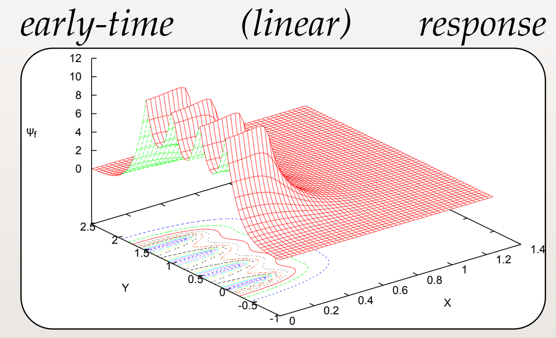
Multiporosity solution (above)

The lower figure on the left shows the variation of the series-based multiporosity model between the pseudo-steady-state W-R double-porosity model, to the transient Kazemi-like solution, for different numbers of terms in the series (for $N > 1000$ the series-based solution is nearly identical to the analytical solution).

The primary benefit of using the simpler pseudo-steady conceptualization, is it results in simpler closed-form solutions. The more realistic transient interporosity Kazemi model must often be solved numerically using finite differences or very complex analytical solutions. Moench [1984] found the W-R approach to only be valid at late time. The multiporosity solution approximates the complex solution using an infinite series of simple solutions, which is more efficient to compute.

Analytic Element Method Solutions

early-time (linear) response

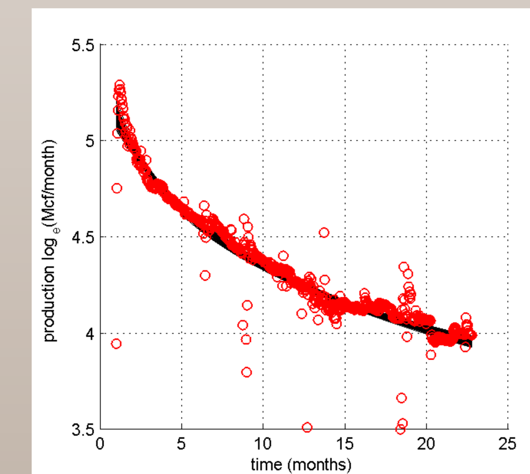


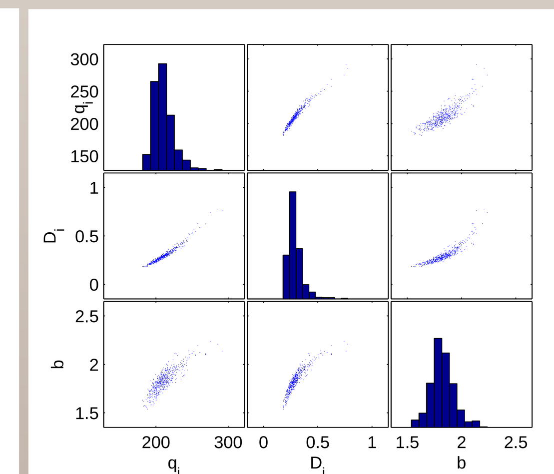
Radially symmetric well test solutions cannot fully represent the complex geometries in horizontally completed wells. As an illustrative example, we use the transient analytical element method [Kuhlman and Neuman, 2009, Bakker and Kuhlman, 2011] to implement the multiporosity solution for a four specified-flux parallel line segments. The surface shows the head distribution at early time, when each fracture zone is essentially isolated (the *linear flow regime* in petroleum engineering).

Unlike radial well-test solutions, analytic elements can represent later stages, when fractures interfere. Although only a square area is contoured there are no outer boundaries enforced on the solution. The analytic element method can be configured with or without outer boundaries. The analytic element method falls between radially symmetric well test solutions and numerical simulators (e.g., TOUGH2) in complexity and ease of use.

Bayesian Analysis

These physically based flow solutions and the widely used Arps model can be fit to shut-in or production data from wells to improve estimates of reservoir and fractures parameters and total reserves. We use the Markov chain Monte Carlo sampler DREAM$_{ZS}$ [ter Braak and Vrugt, 2008] to perform Bayesian analysis.





As a first step in the Bayesian analysis, empirical Arps model ($q = q_i(1 + bD_i t)^{-1/b}$) is fitted to daily gas production data (see left figure). The parameter's posterior distributions are shown as histograms in the right figure, with the bi-variate posterior distributions of parameters on the off-diagonals (showing clear correlation between parameters in this simple model).

We are working to extend the Bayesian approach using the newly developed diffusion-based models and soon-to-be-collected natural tracer data. The posterior probability distributions from one analysis are used as priors information for additional analysis, to show the effects additional information has on parameter estimation and identifiability.

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