Transient particle tracking using LT-AEM

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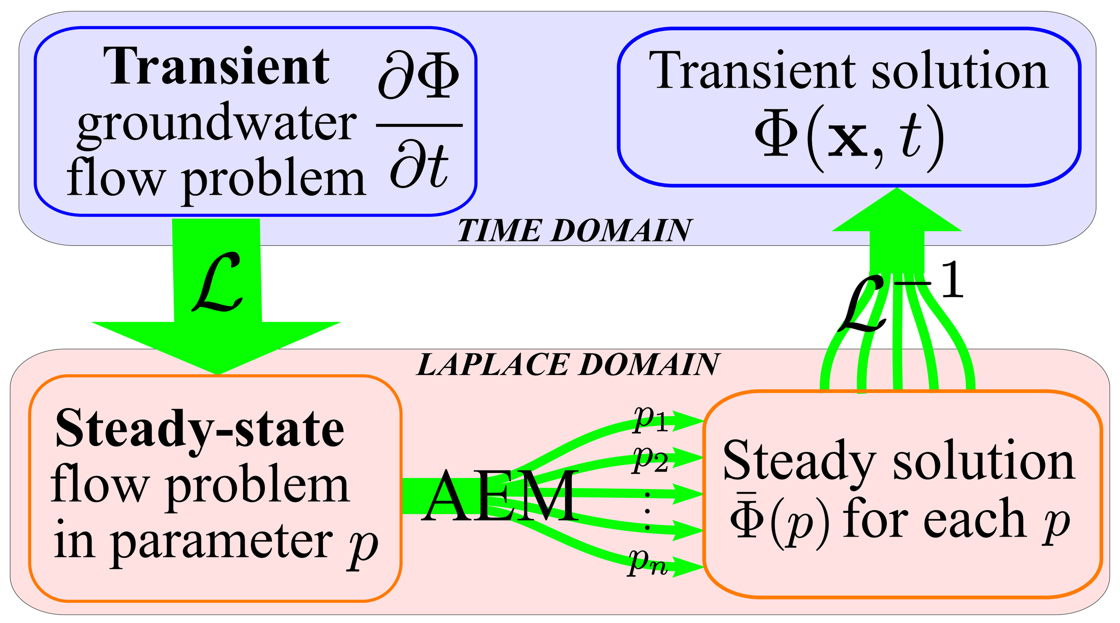
The **Laplace-transform Analytic Element Method** (LT-AEM) is a transient extension of the AEM (begun by Otto Strack at U. Minnesota [3] circa 1980), which utilizes a numerical inverse Laplace transform algorithm ($\mathcal{L}^{-1}$) to compute time-domain solutions from high-accuracy AEM solutions in Laplace space [1, 2].



The Laplace-transformed diffusion equation is the modified Helmholtz equation, when we assume a zero initial condition. Non-zero initial conditions are treated using impulse area sources.

$$\mathcal{L}\left\{ K\nabla^2\Phi = S_s\frac{\partial \Phi}{\partial t}\right\} \quad \longrightarrow \quad \nabla^2\bar{\Phi} - \frac{p}{\alpha}\bar{\Phi} = 0$$

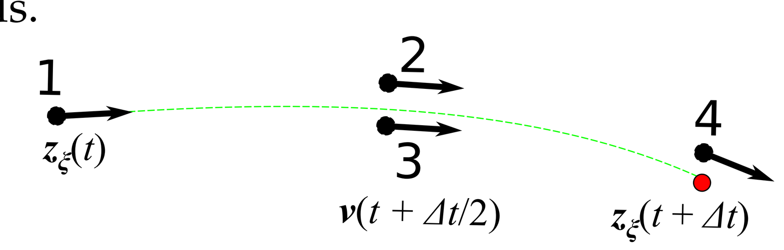
LT-AEM combines analytic solutions so that specified boundary conditions are enforced. Once the coefficients of the LT-AEM solution are known for a given geometry, the solution can be computed at any point in space. This allows very accurate calculation of particle pathlines in transient flow fields.

Particle Integration

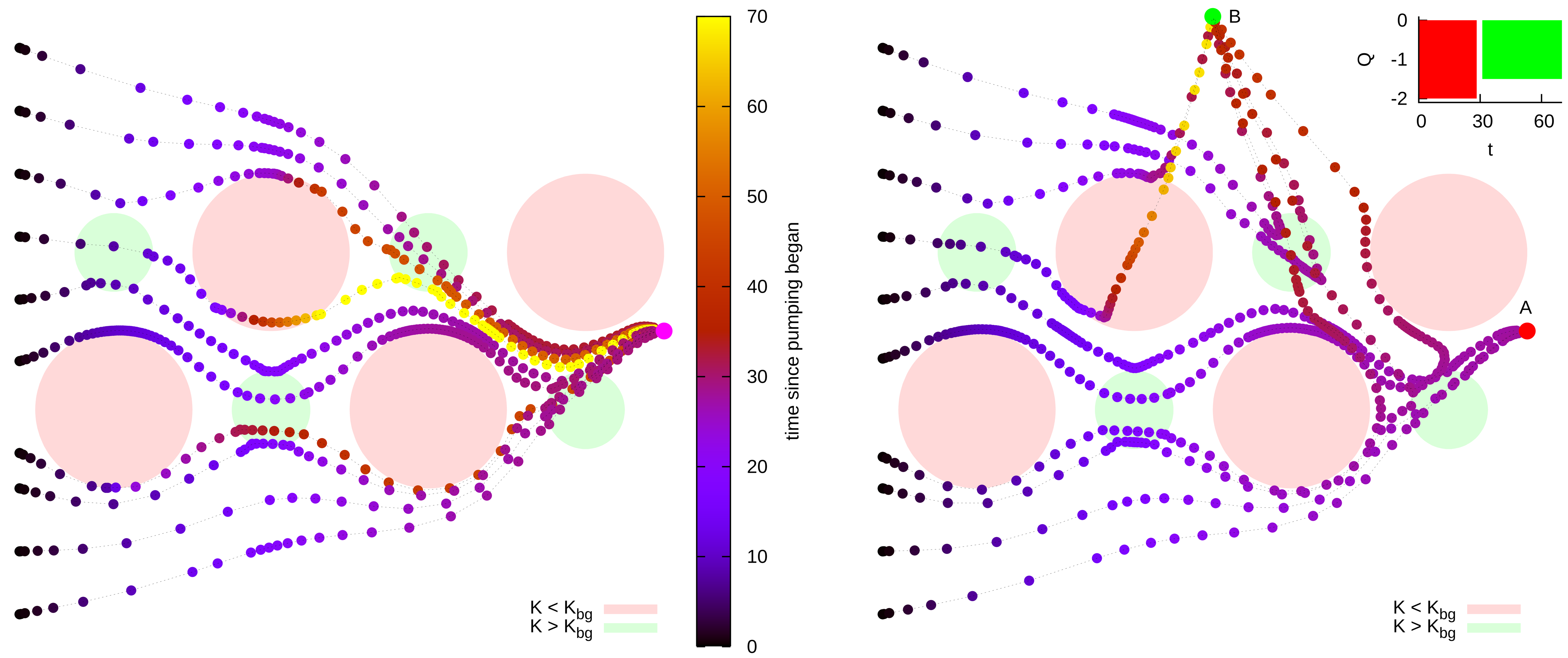
Particle location, $\mathbf{z}(\boldsymbol{\xi}, t-t_0)$ (where $\boldsymbol{\xi}$ is a vector starting location, in the Lagrangian sense), is found in terms of the velocity, $\mathbf{v}(\mathbf{x}, t)$ (where $\mathbf{x}$ is a standard Eulerian coordinate vector), by

$$\frac{\mathrm{d}\mathbf{z}(\boldsymbol{\xi}, t-t_0)}{\mathrm{d}t} = \mathbf{v}(\mathbf{x}, t) \quad \longrightarrow \quad \mathbf{z}_{\boldsymbol{\xi}}(t-t_0) = \mathbf{z}_{\boldsymbol{\xi}}(t_0) + \int_{t_0}^{t} \mathbf{v}(\mathbf{x}, t)$$

An adaptive Runge-Kutta algorithm is used to estimate the integral to a given relative accuracy, by knowing the relative accuracy of two related 4^{th}-order methods.



Runge-Kutta is a type of 4^{th}-order predictor-corrector algorithm, where 4 specific values of velocity (numbered arrows above) are combined to estimate the final particle location at $t + \Delta t$, for a given starting location and time.



These plots show traces of particles released along the left edge of each figure. On the left, a single pumping well operates at a constant rate until the particle are captured, while on the right well A shuts down and well B starts up at $t = 30$. Dot colors indicate time since pumping began. K_{bg} is the background hydraulic conductivity (white areas in the figures).

LT-AEM Benefits

Benefits to using LT-AEM for transient particle tracking include

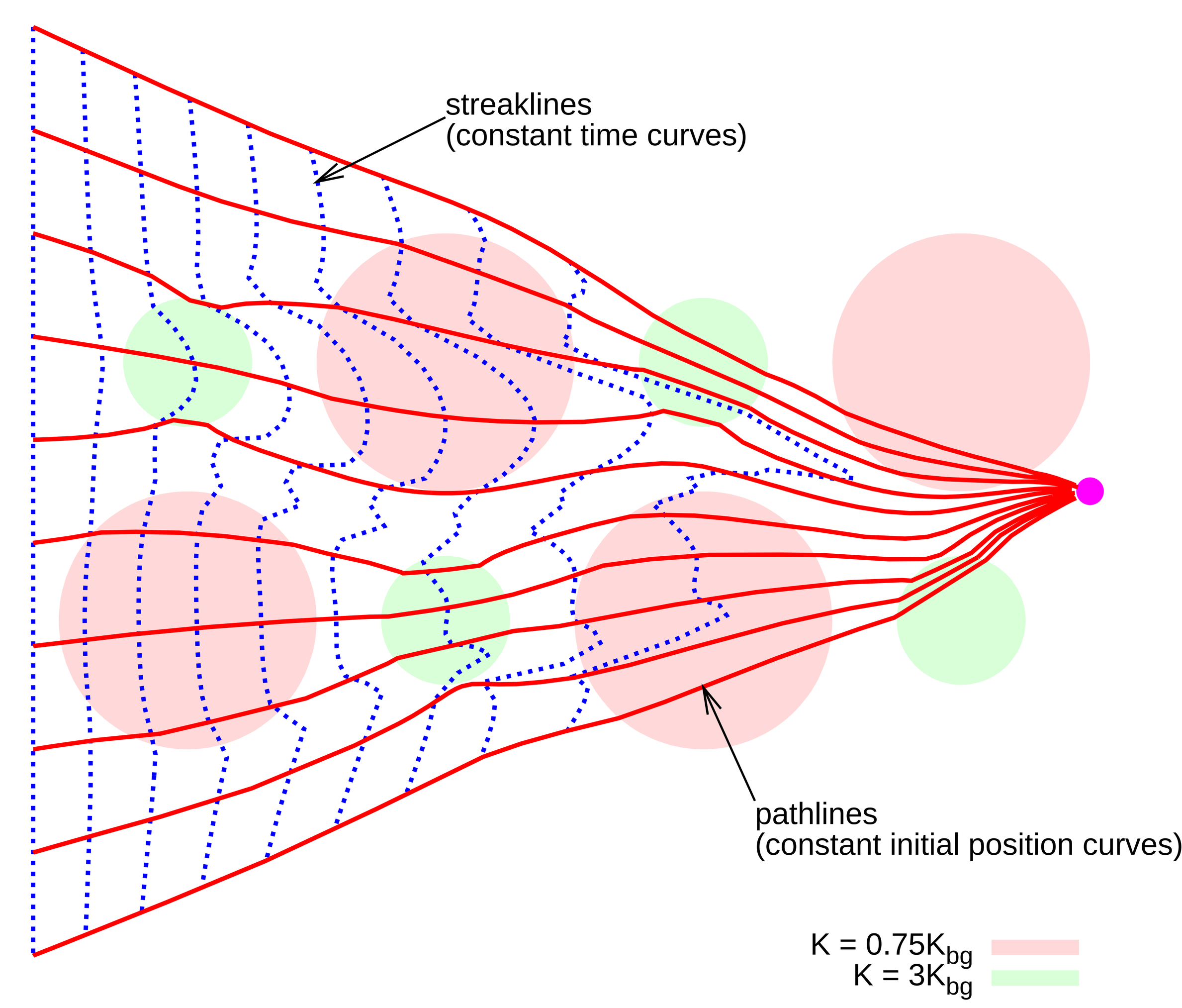
Accuracy: No interpolation; compute results at any point or time

Derivatives: Analytically compute derivatives in Laplace space

∞ **Domain:** No artificial domain boundaries, solution valid $\rightarrow \infty$; computational domain can be either bounded or infinite.

Geometry: Superimpose fundamental elements/geometries

Further applications of LT-AEM are underway to both real-world aquifer test interpretation and the use of Markov chain Monte Carlo inverse models to explore problems where geometry, rather than aquifer parameters, are unknown.



Streaklines and **streamlines** are illustrated using a similar geometry to the above figure, but slightly different aquifer properties. Red circular elements are more permeable here, compared to top figure, as the streaklines are very distorted for the elements with larger K contrast in the top figure.

References

[1] A. Furman and S. P. Neuman. Laplace-transform analytic element solution of transient flow in porous media. *Advances in Water Resources*, 26(12):1229–1237, 2003.

[2] K. L. Kuhlman and S. P. Neuman. Laplace transform analytic element method for transient porous media flow. *Journal of Engineering Mathematics*, in press, 2008.

[3] O. D. L. Strack. *Groundwater Mechanics*. Prentice-Hall, 1989.